## Astrophysical Dynamics

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## 1 Introduction

This talk (set of talks) will explore astrophysical dynamics in a cosmological setting. We first consider the basic physics of perturbation growth and Jean instability; the discussion will then be generalized to include the growth of structure in an expanding universe, self-similarity, and nonlinear growth. One important recent finding is that the universe is observed to be accelerating. The continued acceleration of the universe allows dark matter halos, and hence all cosmic structure, to reach a well-defined asymptotic form. Most of the orbital motion that takes place in the universe arises in these dark matter halos. As a result, we discuss spherical orbits, triaxial orbits, and orbital instabilities. These orbital solutions are then used to describe the formation of both galactic disks and the supermassive black holes that reside in galactic centers. Orbital instabilities are described by Hill's equations (including Mathieu equations) with random parameters; this basic formalism will be reviewed and novel mathematical results will be presented. These same stochastic differential equations can be used to describe the reheating process that occurs at the end of inflation. As a result, this discussion ties together the orbital motion that occurs in future of the universe (primarily for dark matter particles) with the scalar field dynamics that occurs in the ultra-early universe.

## 2 Outline

Perturbation growth in an expanding universe

- cosmic microwave background as initial conditions
- basic analytic equations and power-law growth
- self-similar solutions
- movie showing structure growth from high redshift to the future

The asymptotic form of dark matter halos [1, 2]

- bound structures vs unbound
- phase space portraits
- halo density distributions and halo edges

- mass definitions and asymptotic masses

Orbits in dark matter halos [3, 4, 5, 6, 7]

- basic spherical solutions and orbital elements
- orbits in the inner regime and galactic disk formation

- orbits in point potentials and supermassive black hole formation

- orbits in triaxial potentials and orbit instabilities

- Mathematics of Hill/Mathieu equations [8, 9, 10]
  - basic Floquet theory
  - generalization to random parameters
  - application to astrophysical orbits
  - application to reheating in inflation

For background reading, see references [11] and [12].

## References

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