Measuring the Surface Tension of Water by Light Diffraction on Capillary Waves*

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Surface waves on water may be divided into two regimes. The familiar waves on lakes and oceans, with wavelengths ranging from hundreds of meters to a few centimeters, are called gravity waves. As the name implies, the dominant restoring force is gravity, which returns the disturbed surface of the water to equilibrium. Waves with wavelengths of a few millimeters and less are known as capillary waves. In this regime the dominant restoring force is the surface tension, which tends to minimize the surface area by smoothing out any ripples. Waves with intermediate wavelengths are known as capillary-gravity waves where gravity and surface tension play comparable roles. Because surface waves on fluids are very familiar to most undergraduates and fascinating, the experimental study of capillary waves is of interest and provides an excellent opportunity to study hydrodynamics. Unfortunately, apart from the limitations of the technique, the equipment for surface light scattering is complex and expensive. In this talk we describe a relatively simple method for the generation of standing capillary waves of known frequency in the millimeter wavelength regime and discuss a technique for measuring their wavelength using a laser interferometry. Most texts on hydrodynamics obtain the dispersion relation of capillary-gravity waves by applying the boundary condition at the free surface to the velocity potential [1]. But, for the undergraduates, more accessible derivations is based on conservation of energy [2] which gives well-known dispersion relation for surface waves in the region of deep water

$$\omega^2 = gk + \frac{\sigma k^3}{\rho}, \tag{1}$$

where: ω is angular frequency, g the gravitational acceleration, ρ the fluid density, σ the surface tension and $k = \frac{2\pi}{\lambda}$ wave number with λ as the wavelength. For the capillary waves, because we may neglect the gravity term, in the dispersion relation we obtain

$$\omega^2 = \frac{\sigma k^3}{\rho}. \tag{2}$$

This equation can be recast into the Kelvin equation, which relates the surface tension of a fluid to the frequency and wavelength of the capillary waves,

$$\sigma = \frac{\rho\omega^2}{k^3}, \tag{3}$$

*Work supported in part by the Serbian Ministry of Education and Science, No. 176021.

which is suitable for experimental determination of the surface tension. Apparatus for measuring the wavelength of capillary waves (Figure 1) consists of: signal generator, oscilloscope, speaker, micrometer, liquid, styrofoam container, screen, CCD camera, meter tape, thermometer and cardboard (or a metal plate). Capillary waves are induced by sinusoidal oscillations from the speaker that are transmitted on the plane booster[1] on the surface of the fluid.

Figure 1: Photograph of the experimental set-up.

The surface wave profile acts as a reflection phase grating for incident light. By illuminating the liquid surface with laser light a diffraction picture is produced on the screen (Figure 2).

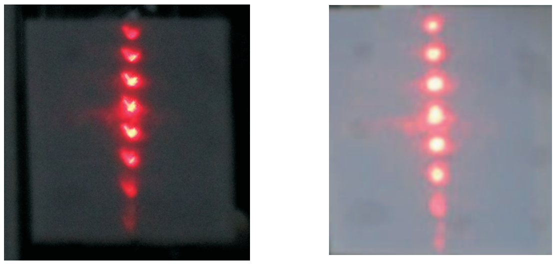

Figure 2: Pictures of diffraction made by the He-Ne laser for 170 and 260 Hz.

By measuring the position of the main peaks, the wavelength of capillary waves is determined $\lambda = d$ (d is a constant of the "grating"). A CCD camera is used for improved accuracy during measuring. Recording of a diffractional image for seven different frequencies with the step of 30 Hz in separate files has been done in the experiment. The data provides an accurate method for determining the surface tension of fluids by the formula (3).

References

[1] L.D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Pergamon Press, New York, 1987

[2] F. Behroozi and A. Perkins, *Am. J. Phys.* **74** (11), (2006) p. 957.

[1] A plane booster is a piece of a razor blade (1x1cm) soldered down to the 3cm long wire which is pasted to the membrane of the speaker.